$FOSLL^*$ Methods for Problems with Irregular Boundaries

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Abstract

The standard $FOSLS$ approach reformulates an elliptic PDE as a first order system $LU = F$, which is solved in a least-square sense. From both the theoretical and the practical point of view it is of great advantage to use the L_2-norm and the standard H^1 finite element spaces. If the solution of the PDE is not smooth enough (i.e. $U \notin H^1$) these goals can not be met.
For a certain class of non smooth solutions, namely the ones arising from discontinous coefficients, it is shown in [1] that solving the dual problem $L^*W = U$ can overcome the above mentioned problems, mainly since $W \in H^1$ holds. This approach is called the $FOSLL^*$ method.
For irregular boundaries (e.g. reentrant corners) or irregular boundary conditions (e.g. some types of mixed Dirichlet and Neumann conditions) a straight forward application of $FOSLL^*$ may yield a solution W of the dual problem, which is not in H^1 as desired.
In this paper we describe an approach that overcomes this difficulty and therefore permits the use of standard finite element spaces in the discretization process. The efficiency of the improved $FOSLL^*$ method is illustrated by several numerical examples.

[1] "First-order system LL^* ($FOSLL^*$): scalar elliptic partial differential equations" *by Z. Cai, T. Manteuffel and S. McCormick*, SIAM J.Num.Anal., submitted