SEMI-CIRCULANT PRECONDITIONING OF ELLIPTIC OPERATORS

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Abstract. This work was motivated by the paper [LH] which discussed the semi-circulant preconditioning of two dimensional convection diffusion equations. The results on the distribution of the eigenvalues of the preconditioned system are extremely interesting. On the other hand these results seemed to contradict a fundamental principle enunciated in [MP]. The basic results of [MP] imply that if one preconditions a discrete elliptic operator L_h with Dirichlet Boundary Conditions by another discrete elliptic operator B_h and one has results such as those of [LH], then B_h must also impose Dirichlet Boundary conditions. However it would appear that the semi-circulant preconditioner has some Dirichlet Boundary Conditions and some periodic boundary conditions. Finally, since part of the attraction of the semi-circulant preconditioner is the fact that this preconditioner is easily inverted, one is led to a study of semi-circulant preconditioners for the uniformly elliptic case.

In this paper we clarify the apparent contradiction between the results of [LH] and [MP]. Simply put, the limiting operator of the difference schemes is *not* elliptic. It is the limiting hyperbolic operator when E, the singular perturbation parameter, goes to zero. Since the solution of the elliptic convection-diffusion equation converges to the solution of this hyperbolic equation, the method and the preconditioning approach are reasonable. Of course, there is a boundary layer in both the solution of the differential equation and the solution of the difference equation. This method does not capture the correct boundary layer. However, that is a small point.

While this clarification is both scientifically important and satisfying it is not the main thrust of our work. Our major effort is directed at the development and exposition of a method for studying these problems which we believe is both more transparent and elementary. Using this approach we both re-obtain the results of [LH] and describe the distribution of the eigenvalues of the semi-circulant preconditioned system in the uniformly elliptic case. As we might expect from the results of [CC] for the full circulant preconditioner, roughly $(\frac{1}{h})$ of these eigenvalues grow like $O(\frac{1}{h})$.

Still, as we shall see, for many of those eigenvalues the coefficients of the growth is quite small. Thus, given the ease of inverting the preconditioner semi-circulant preconditioning may be a useful approach for some problems.

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