Implicit Nonlinearly-Consistent Solutions of the Two-Dimensional Radiation-Hydrodynamics Equations

V. Mousseau*
vmss@lanl.gov

D. Knoll
nol@lanl.gov

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Los Alamos National Laboratory,
Los Alamos, NM 87545

The Radiation-Hydrodynamics equations are a relatively simple equation set (basicly the Euler equations plus nonlinear diffusion added to the energy equation) that allow one to study the dynamics of nonlinearly coupling two distinct physics, compressible fluid flow and nonlinear diffusion. These equations also provide a easy way to develop transients where the radiation time scale is faster than the fluid flow time scale early in the transient and the fluid flow time scale is faster than the radiation time scale late in the transient. This is accomplished by initially introducing a pulse of energy in the radiation field [1] and then allowing the transient to proceed. These pulsed radiation field problems have been studied in one-dimension [2] and this work will solve similar problems in two-dimensions.

Our initial work in one-dimension showed that to achieve a high level of accuracy it was important to solve the fluid equations and the radiation diffusion in an implicit, nonlinearly-consistent fashion. Resent work in other areas [3, 4] indicate that not solving the equations in an implicit and nonlinearly consistent fashion can lead to error terms that prevent accurate solutions at large time steps. We will compare accuracy and efficiency of an implicit and nonlinearly consistent approach to the more traditional operator split approach where the fluid equations are solved separately from the radiation diffusion equations.

In addition to solving the equations in an implicit and nonlinearly consistent fashion we have found that a concept called physics-based preconditioning [5, 6] leads to a very efficient solution algorithm. The basic idea of the physics-based preconditioner for this problem is to precondition the radiation diffusion operator and to resolve the time scales of the fluid flow with the time step (which is required for accuracy). Therefore, to precondition a system with four equations per control volume (continuity, x-momentum, y-momentum, and energy) we will use a preconditioner which only solves a scalar equation which represents only one term (nonlinear diffusion) in the energy equation. The efficiency of this approach will be investigated.

This research will produce the following four studies. The first will be a temporal convergence study which compares the accuracy of a radiation pulse problem using an implicit and nonlinearly consistent method to that of the more traditional operator split approach. The second will be an efficacy study which compares the CPU time for both methods to achieve a fixed level of

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accuracy. The third will be an algorithmic scaling study. Here the physical definition of the problem will be held constant but the number of control volumes will be increased. Both methods will then be compared by determining how the CPU time scales with the increase in problem size. Finally, an analytical study will be done, employing an approximate modified equation analysis, to determine the source of the errors in the operator split approach.

References

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