A Preconditioned Solenoidal Basis Method for Inductance Extraction of VLSI Circuits*

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Abstract

The primary source of signal delay in next generation VLSI circuits is the parasitic resistance, inductance and capacitance of the interconnect joining the circuit elements. Among these, the inductive effect of the interconnects dominates the signal delays at higher frequency. Fast and accurate inductance extraction is becoming an important issue in the design of VLSI circuits. In this paper, we present a solenoidal basis method for solving the linear system arising in inductance extraction. A reduced system is solved in divergence-free subspace by an iterative method. Preconditioners developed for the reduced system have shown promising results on several benchmark problems.

The general technique for inductance extraction discretizes the conductor with a mesh of filaments [1]. The potential drop across a filament is the result of it's own impedance and pairwise mutual impedance with the remaining filaments of the mesh. This gives rise to large, complex and dense linear systems of the form:

$$\begin{bmatrix} \mathbf{Z} & -\mathbf{B} \\ \mathbf{B}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{V} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix}, \tag{1}$$

where $\mathbf{Z} = \mathbf{R} + j\omega\mathbf{M}$ is the impedance matrix, $\mathbf{M}$ is an $n \times n$ dense, symmetric and positive definite matrix representing the inductance of the system, $\mathbf{R}$ is an $n \times n$ diagonal matrix of filament resistances, $\mathbf{I}$ is the vector of branch currents, and $\mathbf{V}$ is the vector of branch potentials. The elements of the inductance matrix $\mathbf{M}$ are given by:

$$\mathbf{M}_{kl} = \frac{\mu}{4\pi}\frac{1}{a_k a_l}\int_{r_k \epsilon V_k}\int_{r_l \epsilon V_l} \frac{\mathbf{u}_k.\mathbf{u}_l}{||\mathbf{r}_k - \mathbf{r}_l||} dV_k dV_l,$$

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The overall system is complex and symmetric, with the complex part restricted to $\mathbf{Z}$. $\mathbf{B}^T$ is the $m \times n$ branch index matrix of filaments and nodes containing two nonzero entries, -1 and 1.

A *solenoidal basis* is a basis for divergence-free functions that automatically satisfy conservation laws such as Kirchoff's current law. A local solenoidal basis can be constructed from unit current flowing along the perimeter of each cell in a uniform 2D mesh. Expressing these currents as vectors, one can construct a full-rank matrix $\mathbf{P}$ which can be used to compute $\mathbf{I}$ via the matrix-vector product: $\mathbf{I} = \mathbf{Px}$ for arbitrary $\mathbf{x} \in R^{(n-m)}$. Restricting the current vector to the subspace of $\mathbf{P}$ automatically satisfies the second constraint in (1). Further simplification yields the following system:

$$\mathbf{P^T Z P x} = \mathbf{P^T F},$$

which is solved via a suitable iterative scheme such as the GMRES method. This approach has been presented in [2].

The success of this approach depends on the ability to precondition the reduced system effectively. This task is complicated by the fact that $\mathbf{Z}$ is available only implicitly through matrix-vector products implemented using hierarchical algorithms such as the Fast Multipole Method (FMM). We present a nearly-optimal preconditioner for the reduced system that demonstrates convergence independent of the problem parameters, namely the mesh size h and the frequency of operation ω. The preconditioning cost can be reduced by using inexpensive variants that do not compromise the rate of convergence significantly. Benchmark experiments have shown that the proposed technique works well in practice and is superior to existing algorithms.

References

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