COMBINING NORMALITY WITH THE FFT TECHNIQUES

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Abstract

A square matrix is perfectly suited to using iterative methods with if an optimal short-term recurrence can be executed with it and matrix-vector multiplications can be performed inexpensively. These are, admittedly, very stringent conditions. The first one renders normal matrices interesting because of the recent introduction of optimal methods for this particular class of matrices [3, 4, 2, 5, 6] while the second one calls, typically, for the FFT ideas. A matrix $A \in \mathbf{C}^{n\times n}$ satisfying

$$AA^* - A^*A = 0 \tag{1}$$

is called normal. Being defined by a second degree algebraic equation, the solution set of (1) is not a linear subspace of $\mathbf{C}^{n\times n}$ and therefore there is no readily available complete parametrization for it. One alternative is to fix a normal matrix and form polynomials in it to have a restricted family of solutions [3]. The set of circulant matrices is obtained in this manner. Although this interpretation is well known, it is not the standard approach. However, thinking in terms of polynomials benefits from unitary invariance, is dimension independent and provides a natural point of view regarding iterative methods in general. Of course, with circulant matrices the FFT techniques can be used as well so that there are matrices satisfying both the prescribed conditions. In this talk we discuss other ways to combine normality with fast matrix-vector multiplication ideas relying, typically, on the FFT techniques. In a fashion similar to the circulant matrices [7, 1], these ideas can be used, for example, to precondition linear systems.

References

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