NORM-WISE AND ITERATION-WISE BOUNDS ON THE CONVERGENCE OF KRYLOV SUBSPACE METHODS RELATIVE TO GMRES

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ABSTRACT. We discuss existing and new residual norm convergence results for non-Hermitian problems for various Krylov subspace methods relative to the convergence of (full) GMRES. It is well-known that for most methods we cannot give small or even finite bounds on the difference between the residual and the GMRES residual that hold at every iteration. Therefore, we study their relative convergence in two alternative ways. First, we relate the convergence of various methods to that of GMRES norm-wise per iteration using bounds that vary per iteration, but depend on certain computable values. Second, we use results from our first analysis to study the relative convergence iteration-wise per norm or for a range of norms (tolerances). That is, how many additional iterations do methods need to converge to a given tolerance above those for GMRES. We will discuss methods based on the two-sided Lanczos process, such as BiCG and QMR, and methods based on 'variants' of GMRES, restarted, truncated, and nested methods. We will also point out various open problems.

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