Preconditioning Newton-Krylov Methods in 2D Hall MHD

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Abstract

We present results from our research on Jacobian-free Newton-Krylov (JFNK) methods applied to the time-dependent 2D Hall magnetohydrodynamics (MHD) equations. MHD is a fluid description of the plasma state. While plasma is made up of independent (but coupled) ion and electron species, the standard MHD description of a plasma only includes ion time and length scales (one-fluid model). In Hall MHD, some electron physical terms are retained in the fluid description (two-fluid model), which increases the stiffness and nonlinearity of the resulting PDE system.

The standard MHD description is by itself stiff, with dynamical time scales orders of magnitude slower than normal mode (Alfvén wave) time scales. We have already developed a successful Newton-Krylov solver for such problems on the basis of the physics-based preconditioning strategy [1,2]. Here, we extend this work by including electron Hall physics, of relevance in the study of magnetic reconnection is space and laboratory plasmas [3]. The new set of equations supports the so-called whistler wave, whose dispersion relation is quadratic in the wavenumber k (dispersive). As a consequence, the whistler wave group velocity decreases linearly with the mesh spacing. In an explicit numerical formulation, this property results in a stringent CFL limit $\Delta t_{CFL} \propto \Delta x^2$, which severely limits the application of explicit techniques in extended MHD. An implicit formulation removes the CFL constraint, but, in the context of physics-based preconditioned Newton-Krylov techniques, the dispersive nature of the whistler wave manifests itself in a fourth-order, anisotropic, symmetric positive definite parabolic operator of the form $[I + \alpha(\vec{B} \cdot \nabla)\nabla^2(\vec{B} \cdot \nabla)]$, where $\vec{B}$ is the magnetic field, and $\alpha \propto \Delta t^2$.

The inversion of this operator in arbitrary magnetic field configurations using approximate multigrid (MG) techniques is a crucial step in the effectiveness of the preconditioner. However, this step is particularly challenging given the nature of the operator. Additionally, we desire a matrix-light MG implementation in which the MG algorithm is entirely expressed in the form of matrix-vector products, and only the diagonal of the matrix need be known for smoothing purposes. The availability of an efficient and effective smoother is particularly relevant to the success of this approach. Here, we have developed an efficient nearest-neighbors weighed-Jacobi smoother for this application, of straightforward matrix-light implementation.

Our presentation will focus on the details of the construction of a physics-based preconditioner for a 2D Hall MHD model, on the MG treatment of the fourth-order anisotropic operator (including discretization and smoothing issues), and on the performance of the resulting preconditioning strategy in an implicit JFNK implementation.

References

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