An algebraic multilevel method for sparse approximate inverses based on norm minimization

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Abstract

We present an algebraic multilevel method that is based on sparse approximate inverse matrices. The approach is based on the observation that sparse approximate inverses based on norm minimization [4,3,2] can easily be adapted such that they approximate the operator quite well on a large subspace. A natural consequence is to augment the sparse approximate inverse with a correction term of smaller rank. This is an alternative to allowing a denser nonzero pattern for the sparse approximate inverse. We will present techniques for the construction of the correction term. As one part of the construction in the symmetric case, one can use an approximate QR decomposition to detect certain columns of the residual matrix for the correction term [1]. However, the construction can be generalized to the unsymmetric case. We will present several examples to illustrate the effectiveness of this new approach.

Keywords. algebraic multilevel methods, sparse approximate inverse matrices, iterative methods, preconditioning, partial differential equations.

[1] M. Bollhöfer and V. Mehrmann. Algebraic multilevel methods and sparse approximate inverses. *SIAM J. Matrix Anal. Appl.* to appear.

[2] M. J. Grote and T. Huckle. Parallel preconditioning with sparse approximate inverses. *SIAM J. Sci. Comput.*, 18(3):838–853, 1997.

[3] I. E. Kaporin. New convergence results and preconditioning strategies for the conjugate gradient method. *Numer. Lin. Alg. w. Appl.*, 1(2):179–210, 1994.

[4] L. Kolotilina and A. Yeremin. Factorized sparse approximate inverse preconditionings. I. Theory. *SIAM J. Matrix Anal. Appl.*, 14:45–58, 1993.