Inverse Iteration and inexact solves

J. Berns-Müller, I.G. Graham and A.Spence

Inverse iteration using exact linear solves is a popular method for matrix eigenvalue problems with well understood convergence behaviour. However for large sparse matrices, say coming from discretised PDE's, exact solves are often not feasible. In this talk we consider the effect on the convergence behaviour by replacing the exact solves by an inexact iterative solver.

At each step of inverse iteration the system

$$(A - \sigma^i I)y^i = x^i, \tag{1}$$

needs to be solved. If, instead of solving (1) exactly, y^i is found approximately in the sense that

$$r^i = x^i - (A - \sigma^i I)y^i,$$

where $\| r^i \| \leq \tau^i$, for some τ^i.

In the first part of the talk we describe an error analysis similar to that in Smit & Paardekooper (*Lin. Alg. Appl., vol 287, pp.40-425, (1999)*) and Golub & Ye (*BIT, vol. 40, pp.671-684, (2000)*). We consider first arbitrary inexact solvers and show how convergence depends on the two parameters σ^i and τ^i. Further, under certain conditions on σ^i and the iterative solver, we show how inexact inverse iteration regains the convergence speed achieved using exact solvers. For example, for A symmetric, we discuss in detail the case when σ^i is chosen to be the Rayleigh Quotient. We give numerical examples to illustrate our theory. In the second part of the talk we extend our results to nonsymmetric and generalised eigenvalue problems.