A Taxonomy of Stabilized FEM for the Stokes Problem: Preliminary Studies on Stability, Accuracy and Convergence

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Abstract

It is well known that Finite element spaces used by mixed Galerkin methods in the solution of the Stokes equations must satisfy the restrictive inf-sup compatibility condition, if stable and accurate approximations are desired. This inf-sup condition can be avoided by regularization of the mixed variational problem; this leads to a class of *stabilized Galerkin* methods, and weak variational equations that are well-posed for any conforming choice of the finite element spaces. However, stabilization leads to a dichotomy in the type of the variational equations - they are either weakly coercive and possibly symmetric or strongly coercive and non-symmetric. In addition, some approaches lead to unconditional stabilization while others give methods whose stability depends on proper choices of regularizing parameters and/or implementation. As a result, finite element algebraic problems generated by stabilized methods range from symmetric indefinite systems that are unconditionally stable to non-symmetric, conditionally positive definite problems.

Such wide and varied properties of the discrete problems make the choice of stabilizing parameters and iterative solvers critical to the overall success and accuracy of stabilized methods. This talk presents a comparative study of stabilized methods for the Stokes equations. We begin with a general framework, based on a penalized Lagrangian formulation, which can be used to generate the methods and to reveal their taxonomy. Then we focus on the impact of regularization parameters upon convergence and stability of finite element approximations. Finally we compare performance of iterative solvers for the two alternative types of linear systems associated with each method and study how different choices of regularization parameters affect these solvers.

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