On Krylov subspace acceleration of linear and nonlinear multigrid schemes for rotating flow problems

C.W. Oosterlee * and T. Washio ‡

Short Abstract:

This talk deals with the combination of two solution methods: multigrid and GMRES [7]. The generality and parallelizability of the combined solution method are presented by solving systems of nonlinear PDEs, like the incompressible Navier-Stokes equations. As the 'preconditioner' for a nonlinear Krylov method, the Full Approximation Storage (FAS) scheme [1], a nonlinear multigrid method is used. The nonlinear Krylov acceleration is also applied on coarse grids, so that rotating flow problems discretized with a higher order upwind scheme can be solved efficiently.

Extended Abstract:

In the search for efficient solvers that are generally applicable, we consider multigrid as a preconditioner for Krylov subspace methods. Although multigrid solution methods are $O(N)$ solvers, it is not always easy to choose the optimal components for difficult problems. The *optimal* multigrid method for systems of equations including combinations of complicated factors, like convection-dominance, anisotropies, nonlinearities or non M-matrix properties (in multiblock situations) is often difficult to design. It is also not trivial to construct *robust* multigrid solvers for large classes of problems. We aim to increase by Krylov subspace acceleration the class of problems for which proven efficient multigrid solvers exist. In the case of an $O(N)$ multigrid solver, we try to improve its convergence rate by the acceleration or to reduce the work per iteration, for example by reducing the number of smoothing steps. For large systems of equations the acceleration technique is, in general, cheaper in CPU time than a smoothing iteration on the finest grid. In the first part of the talk, we concentrate on linear problems in order to illustrate the idea of the combination and we accelerate linear multigrid with GMRES [7].

It is possible to generalize the combined solution method to systems of equations and nonlinear problems, as we will see in the second part of this talk. In a natural way, a multigrid method can be constructed for nonlinear problems, see [1], [4], so that linear and nonlinear problems can be handled by the one and the same method. It is shown that the same principle holds for the Krylov subspace acceleration. If we switch from GMRES to FGMRES [8], it is not so difficult to generalize the method to nonlinear situations. With the combined solution method many different nonlinear problems can be solved, as we showed in [9]. In this talk we concentrate on a class of problems, that are particularly difficult to solve with standard multigrid methods, the class of rotating flow problems with dominant convection. We will explain the difficulties of rotating flow problems in some detail. In [2] the same type of problems are treated and remedies are given for improving the multigrid convergence rate for the first order upwind discretization of the convection terms. We investigate the effect of the Krylov subspace acceleration on first and second order order upwind discretizations, where we emphasize finding efficient solution methods for the latter discretization. An interesting topic for making the combined

*GMD, Institute for Algorithms and Scientific Computing, D-53754 Sankt Augustin, Germany (email: oosterlee @ gmd.de)

‡C&C Research Laboratories, NEC Europe Ltd., D-53757 Sankt Augustin, Germany (email: washio @ ccrl-nece.technopark.gmd.de)

method feasible for solving realistic 3D CFD problems is the reduction of storage. Much storage is necessary when a large Krylov subspace is needed for searching an accelerated solution. We discuss a possibility and show results of the use of the Krylov acceleration also on coarser grids. On coarse grids much less storage is needed for storing a Krylov subspace.

Another advantage of the combined solver lies in its parallelizability. Both parts can be parallelized in a standard way on the basis of grid partitioning [5]. Parallel numerical convergence results with the nonlinear combined solution method are presented for systems of the incompressible Navier-Stokes equations, where the discretization is based on the primitive variables. The discretization of the Navier-Stokes equations is based on flux-difference splitting (see [3]) with higher order upwind κ-discretizations [6] for the convective terms, so that the resulting discretization is second order accurate. In such flow situations we are dealing exactly with the difficult phenomena convection-dominance, anisotropies, nonlinearities and non M-matrix properties mentioned above.

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