Convergence and Parallelization of a Preconditioned Algorithm for Eigenvalues

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Two well known subspace methods for large symmetric eigenvalue problems are the Lanczos and Davidson algorithms. Both methods solve the eigenvalue problem $Au = \lambda u$ by constructing an orthonormal basis $V_k = \{v_1, \ldots, v_k\}$, at each k^{th} iteration step, and then find an approximation for the eigenvector u of A by using a vector u_k from the subspace spanned by V_k. The Lanczos algorithm has been more popular than Davidson, partly due to the fact that for Lanczos each projected matrix S_k is tridiagonal, while the Davidson algorithm defines a dense matrix S_k on the subspace. On the other hand, a preconditioner can be incorporated into the Davidson algorithm to lower the number of iterations. To improve efficiency, the author has shown that there is a recursive relationship between the subspaces involving arrowhead matrices. In addition, new algorithms for finding eigenvalues of arrowhead matrices were shown in [1].

A proof of convergence (but without a rate estimate) is given in Crouzeix, Philippe and Sadkane [2] for Davidson. The convergence rate was first presented in [3] and is stated by the following theorem.

Theorem 1 *Let M be the preconditioner in the Davidson algorithm, and P the orthogonal projection onto $ker(A - \lambda I)^{\perp}$. Suppose that A and M are symmetric positive definite. If*

$$\|P - PMP(A - \lambda I)\|_2 \leq \sigma < 1,$$

then for almost any starting value x_1, the convergence of the eigenvalue estimates $\widehat{\lambda}_k$ converge to λ ultimately geometrically with convergence factor σ^2, and the angle between the computed eigenvector and the exact eigenspace goes to zero ultimately geometrically with convergence factor σ.

In [3] a variation of the Davidson algorithm for finding a group of extreme eigenvalues was presented. The parallel implementation for this algorithm was developed in collaboration with L. Borges and is presented in [4]. Our numerical results are obtained with a variety of preconditioners.

References

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[3] S. Oliveira. A convergence proof of an iterative subspace method for eigenvalues problem. In F. Cucker and M. Shub, editors, *Foundations of Computational Mathematics Selected Papers*, pages 316–325. Springer, January 1997.

[4] L. Borges and S. Oliveira. A parallel davidson-type algorithm for several eigenvalues. Submitted.